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THEORY OF DIELECTRIC OPTICAL WAVEGUIDE AND BENDING LOSSES.(U)

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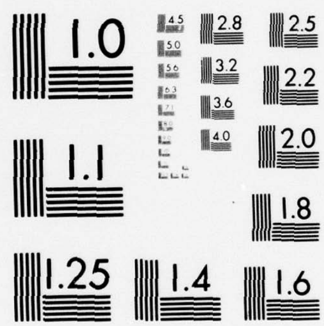

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THEORY OF DIELECTRIC OPTICAL WAVEGUIDE AND BENDING LOSSES

by

L. D. Hutcheson
Research Department

May 1977

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This report contains a review of those theoretical aspects of integrated optics which are pertinent to the bending loss studies under way at NWC. Guided modes and their solutions and an introduction to the bending losses in optical dielectric waveguides are presented. The density of optical components on an integrated optical circuit is limited by the loss due to bends in the optical waveguides. This is a report on a preliminary study and is released at the working level for information only.

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INTRODUCTION

Integrated optics (IO) involves the technology of guiding and manipulating optical waves in dielectric waveguides, the generation and detection of optical waves, and the coupling of optical waves into and out of IO circuits. The simplest form of optical waveguides is a dielectric slab. By studying the properties of slab waveguides, an understanding of more complicated waveguides is possible.

A planar dielectric slab waveguide is shown in Figure 1. The waveguide region has an index of refraction, n_1, with surrounding media having indices of n_0 and n_2. For waveguiding, the following criteria must be met:

$$n_1 > n_2 \geq n_0 \quad . \tag{1}$$

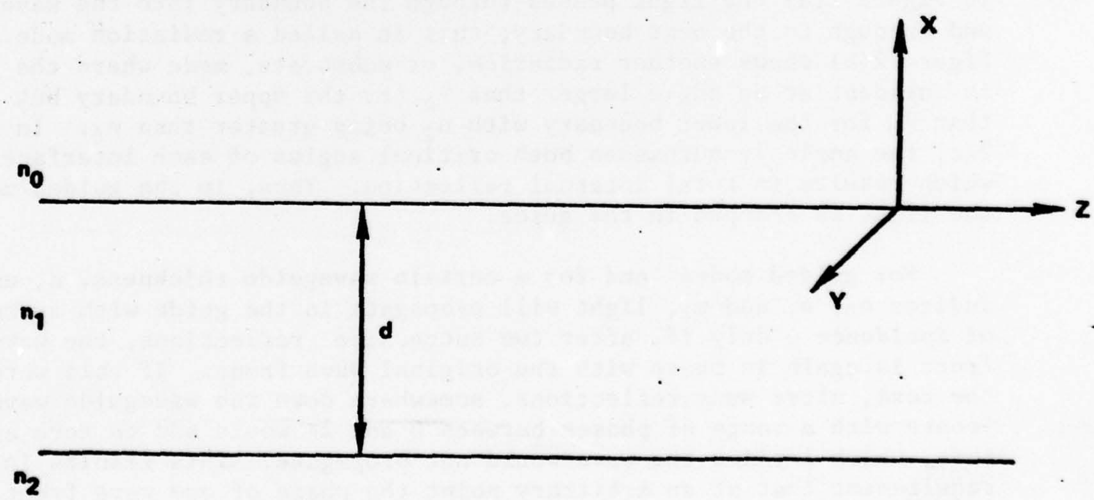

FIGURE 1. A Planar Dielectric Slab Waveguide.

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[1] Naval Weapons Center. *Integrated Optics*, by L. D. Hutcheson. China Lake, Calif., NWC, December 1975. (NWC TM 2592, UNCLASSIFIED.)

If $n_2 = n_0$, then it is called a symmetric waveguide. If $n_2 \neq n_0$, then there exists an asymmetric waveguide. Of course, the modal solutions which satisfy the boundary conditions are simpler for the symmetric case than for the asymmetric case. It is also interesting to note that for the symmetric waveguide there is no cutoff frequency[2] which implies that the lowest order mode can propagate at very low frequencies. However, there does exist a low frequency cutoff for all the modes of the asymmetric waveguide.

There are two different types of electromagnetic modes in dielectric waveguides, guided modes and radiation modes. Figure 2 distinguishes between these two different types of modes. To obtain total internal reflection, the incident angle, θ_1, must be greater than the critical angle, θ_c, for each interface. For interface 0,1

$$\theta_1 > \theta_c = \sin^{-1}\left[\frac{n_0}{n_1}\right]. \tag{2}$$

For interface 1,2

$$\theta_1 > \theta_c = \sin^{-1}\left[\frac{n_2}{n_1}\right]. \tag{3}$$

In Figure 2(a) the light passes through the boundary into the waveguide and through to the next boundary; this is called a radiation mode. Figure 2(b) shows another radiation, or substrate, mode where the light is incident at an angle larger than θ_c for the upper boundary but smaller than θ_c for the lower boundary with n_2 being greater than n_0. In Figure 2(c) the angle θ_1 surpasses both critical angles of each interface, which results in total internal reflection. Thus, in the guided mode the light is trapped in the guide.

For guided modes[3] and for a certain waveguide thickness, d, and indices n_0, n_1 and n_2, light will propagate in the guide with an angle of incidence θ only if, after two successive reflections, the wave front is again in phase with the original wave front. If this were not the case, after many reflections, somewhere down the waveguide wave fronts with a range of phases between 0 and 2π would add to zero amplitude, which implies the wave would not propagate. This results in the requirement that at an arbitrary point the phase of one wave front obtained from another by two successive reflections must equal the phase of the second wave front at the same point, or must differ by a multiple

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[2] D. Marcuse. *Theory of Dielectric Optical Waveguides*. New York, Academic Press, 1974.

[3] R. A. Andrews. *Optical Waveguides and Integrated Optics Technology*, Naval Research Laboratory Report 7291 (August 1971), UNCLASSIFIED.

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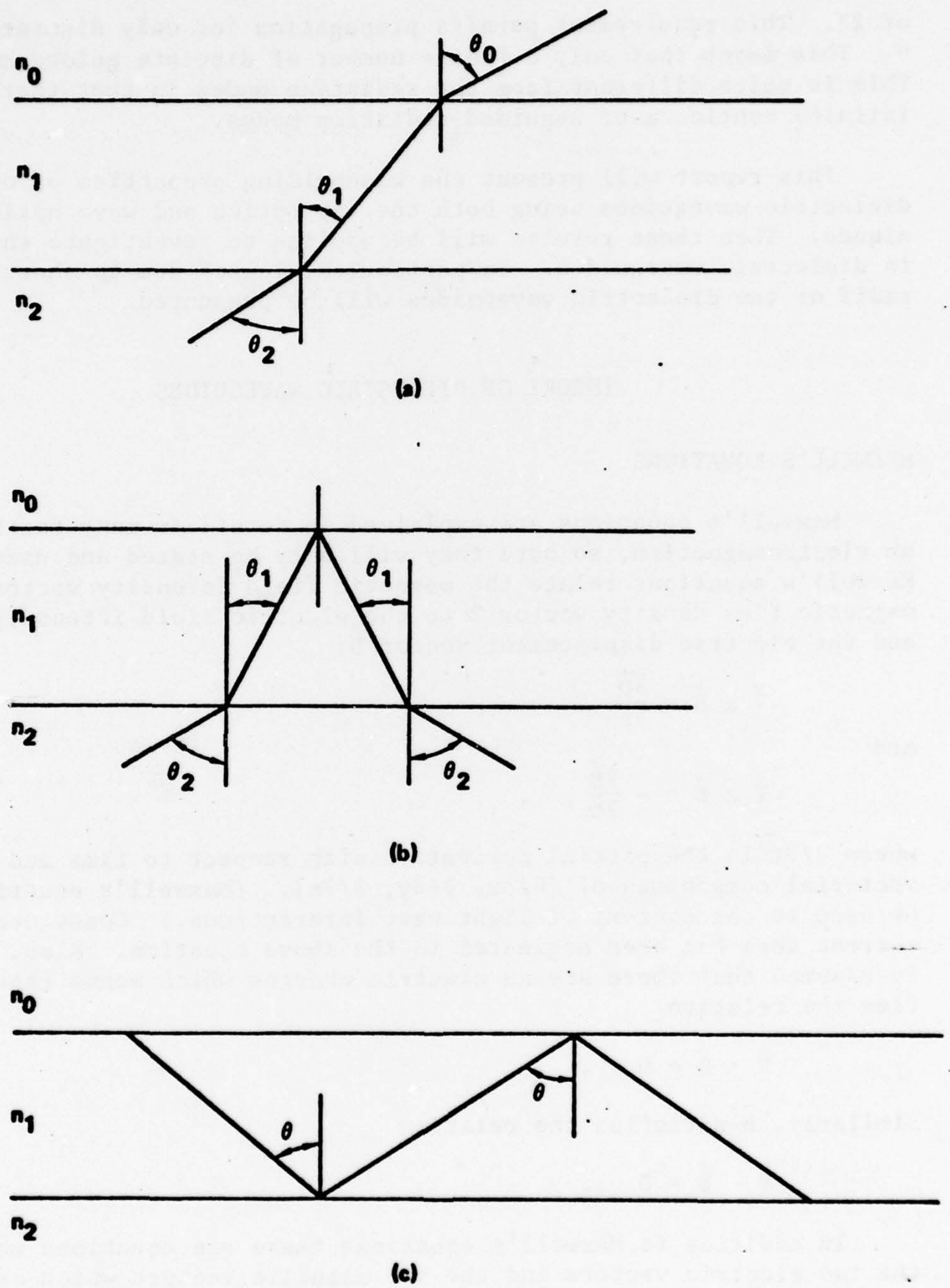

FIGURE 2. Ray Diagrams Illustrating (a) Radiation Modes, (b) Substrate (Radiation) Modes, and (c) Guided Modes.

of 2π. This requirement permits propagation for only discrete values of θ. This means that only a finite number of discrete guided modes exists. This is quite different from the radiation modes in that there exists an infinite continuum of unguided radiation modes.

This report will present the waveguiding properties of optical dielectric waveguides using both the ray optics and wave optics techniques. Then these results will be applied to investigate the losses in dielectric waveguides. In particular, losses due to short bending radii of the dielectric waveguides will be presented.

THEORY OF DIELECTRIC WAVEGUIDES

MAXWELL'S EQUATIONS

Maxwell's equations are explained in detail in many textbooks[4,5] on electromagnetism, so here they will only be stated and used as needed. Maxwell's equations relate the magnetic field intensity vector $\vec{H}$ and the magnetic flux density vector $\vec{B}$ to the electric field intensity vector $\vec{E}$ and the electric displacement vector $\vec{D}$:

$$\vec{\nabla} \times \vec{H} = \frac{\partial \vec{D}}{\partial t} \tag{4}$$

and

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad , \tag{5}$$

where $\partial/\partial t$ is the partial derivative with respect to time and $\vec{\nabla}$ has the vectorial components of $[\partial/\partial x, \partial/\partial y, \partial/\partial z]$. (Maxwell's equations will be used in the context of light wave interactions.) Consequently, the current term has been neglected in the above equation. Also, it will be assumed that there are no electric charges which means that $\vec{D}$ satisfies the relation

$$\vec{\nabla} \cdot \vec{D} = 0 \quad . \tag{6}$$

Similarly, $\vec{B}$ satisfies the relation

$$\vec{\nabla} \cdot \vec{B} = 0 \quad . \tag{7}$$

In addition to Maxwell's equations there are equations which relate the two electric vectors and the two magnetic vectors which are, in general, material dependent:

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[4] C. H. Durney and C. C. Johnson. *Introduction to Modern Electromagnetics*. New York, McGraw-Hill, 1969.

[5] J. A. Stratton. *Electromagnetic Theory*. New York, McGraw-Hill, 1941.

6

$$\vec{D} = \tilde{\varepsilon} \cdot \vec{E} \tag{8}$$

and

$$\vec{B} = \tilde{\mu} \cdot \vec{H} \quad , \tag{9}$$

where $\tilde{\varepsilon}$ and $\tilde{\mu}$ are tensors representing the material permittivity and permeability, respectively. If $\tilde{\varepsilon}$ and $\tilde{\mu}$ are functions of positions in the medium then the medium is called inhomogeneous. If there is no position dependence, the medium is called homogeneous. On the other hand, if $\tilde{\varepsilon}$ and $\tilde{\mu}$ are tensors, the medium is termed anisotropic while an isotropic medium is one in which $\tilde{\varepsilon}$ and $\tilde{\mu}$ are scalar quantities. Therefore, the most general type of medium is anisotropic and inhomogeneous. However, for many cases of practical interest, the medium can be approximated to be isotropic and homogeneous, which is what will be assumed in the following analysis.

To obtain the useful wave equation one must manipulate Maxwell's relations until an equation results which depends only on $\vec{E}$ or $\vec{H}$. To make the analysis simpler, without losing generality, one can assume one of the guided dimensions to be much larger than the other guided dimension. In the waveguide shown in Figure 1, the y-dimension is made much larger than the x-direction which yields a planar waveguide and in Maxwell's equations one can take

$$\frac{\partial}{\partial y} = 0 \quad . \tag{10}$$

The planar waveguide is the same solution as the channel waveguide with one dimension much larger than the other.[6] This actually does not restrict the generality of the modal analysis since the coordinate system can always be rotated until this condition is satisfied for any given mode.

Before one can fully understand the interactions of guided waves a knowledge of the properties of guided modes must be obtained. A concise definition of the term "mode" is difficult to make. A mode can be regarded as an eigensolution of Maxwell's equations belonging to a particular eigenvalue and satisfying all the boundary conditions of the problem.[7] The modes of a dielectric slab waveguide are classified as transverse electric (TE) and transverse magnetic (TM) modes. A TE mode

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[6] E. A. J. Marcatili. "Dielectric Rectangular Waveguide and Directional Coupler for Integrated Optics," BELL SYST TECH J, Vol. 48 (September 1969), pp. 2071-2102.

[7] D. Marcuse. *Light Transmission Optics*. New York, Van Nostrand Reinhold, 1972.

means that the electric field component in the direction of wave propagation does not exist. A TM mode has no component of the magnetic field in the longitudinal direction. Since the TE and TM modes have different components they must be studied separately.

TE-GUIDED MODES

Transverse electric modes only have three field components which are E_y, H_x, and H_z. In a linear and isotropic material Maxwell's Equations 4 and 5 become, after using relations 8 and 9,

$$\vec{\nabla} \times \vec{H} = \varepsilon_0 n^2 \frac{\partial \vec{E}}{\partial t} \tag{11}$$

and

$$\vec{\nabla} \times \vec{E} = -\mu_0 \frac{\partial \vec{H}}{\partial t} \quad , \tag{12}$$

with

$$\vec{\nabla} = \hat{x} \frac{\partial}{\partial x} + \hat{y} \frac{\partial}{\partial y} + \hat{z} \frac{\partial}{\partial z} \quad , \tag{13}$$

where $\hat{x}$, $\hat{y}$ and $\hat{z}$ are unit vectors in the x, y and z directions, respectively, and n is the index of refraction of the medium. Assuming a field of the form

$$e^{i(\omega t - \beta z)} \quad , \tag{14}$$

where β is the propagation constant along the axis of waveguide and using relation 10, Maxwell's equations yield

$$-i\beta H_x - \frac{\partial H_z}{\partial x} = i\omega\varepsilon_0 n^2 E_y \quad , \tag{15}$$

$$i\beta E_y = -i\omega\mu_0 H_x \quad , \tag{16}$$

$$\frac{\partial E_y}{\partial x} = -i\omega\mu_0 H_z \quad . \tag{17}$$

Putting H_x and H_z in terms of E_y, we get

$$H_x = -\frac{\beta}{\omega\mu_0} E_y \tag{18}$$

$$H_z = \frac{i}{\omega\mu_0} \frac{\partial E_y}{\partial x} \quad . \tag{19}$$

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Substituting Equations 18 and 19 into 15 yields the familiar wave equation in terms of E_y,

$$\frac{\partial^2 E_y}{\partial x^2} + (k^2n^2 - \beta^2)\, E_y = 0 \quad , \tag{20}$$

where $k^2 = \omega^2\mu_0\varepsilon$. Therefore, by solving the one-dimensional-wave Equation 20 for E_y, the magnetic field components, H_x and H_z, are obtained directly from Equations 18 and 19.

Before trying to solve the wave equation in each of the three regions, a physical picture of the modes can be obtained by seeing what happens when the propagation constant, β, at some fixed frequency, takes on different values. Refer to Figure 1 to see the relationship between the three regions and the indices. Also, the waveguide is assumed to be asymmetric with $n_1 > n_2 > n_0$. Therefore, the wave equation is different for each of the three regions since n in Equation 20 takes on n_0, n_1 and n_2. Clearly, if $\beta > kn_1$, which is the largest β possible, and[8] since $1/E\ \partial^2E/\partial x^2 > 0$, then E must be exponential in all three regions. The boundary conditions at both interfaces must be matched which means that the field increases to infinity exponentially away from the waveguide. This says that for this β the solution is not physically realizable and is not a real wave.

To obtain a guided mode solution the propagation constant must satisfy kn_0, $kn_2 < \beta < kn_1$ which yields a *sinusoidal* solution in region one and an exponential solution in the other two regions. This, of course, can be achieved only if $n_1 > n_2$, n_0, which further supports the criterion that the waveguiding layer must have the largest index as was shown in Equation 1. The radiation modes exist for $kn_0 < \beta < kn_2$, which is the substrate radiation mode. Also, for $0 < \beta < kn_0$ radiation modes exist in both surrounding media since the field is sinusoidal in all three regions.

When solving the wave equation, the solutions must satisfy the boundary conditions at the two dielectric interfaces at x = 0 and x = -d. The boundary conditions are such that the tangential E and H fields must be continuous at the dielectric interfaces. This means that both E_y and H_z must be continuous at x = 0 and x = -d. In addition to this requirement, to obtain a physically realizable solution the electric field, E_y, must be zero at $x = \pm\infty$. Since the term $k^2n^2 - \beta^2$ takes on different values in each of the three regions, it will be simpler to define three new variables:

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[8] A. Yariv. *Quantum Electronics*, 2nd Edition. New York, John Wiley and Sons, Inc., 1975.

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$$\alpha^2 = k^2 n_1^2 - \beta^2 \quad , \tag{21}$$

$$\gamma^2 = \beta^2 - k^2 n_2^2 \quad , \tag{22}$$

$$\delta^2 = \beta^2 - k^2 n_0^2 \quad . \tag{23}$$

These definitions provide for α, γ and δ being real and the solution to the wave equation can be assumed to be of the form

$$E_y = \begin{cases} Ae^{-\delta x} & 0 \le x \\ Be^{i\alpha x} + Ce^{-i\alpha x} & -d \le x \le 0 \\ De^{\gamma(x+d)} & -d \ge x \quad . \end{cases} \tag{24}$$

The constants A, B, C and D are determined through the use of the boundary conditions at x = 0 and x = -d, upon which applying the continuity of E_y across the boundary yields

$$A = B + C \tag{25}$$

and

$$D = Be^{-i\alpha d} + Ce^{i\alpha d} \quad , \tag{26}$$

and since H_z must also have its tangential components equal across the boundaries the following additional equations are obtained:

$$-\delta A = i\alpha B - i\alpha C \tag{27}$$

and

$$\delta D = i\alpha Be^{-i\alpha d} - i\alpha Ce^{i\alpha d} \quad . \tag{28}$$

Now we have four equations and the four unknown constants (A, B, C and D). Solving the four equations by the method of determinants results in the following relation:

$$\alpha d = \arctan\left[\frac{\gamma}{\alpha}\right] + \arctan\left[\frac{\delta}{\alpha}\right] \quad . \tag{29}$$

The fact that Equation 29 is periodic with a period π, since the tan is periodic in π, says that relation 29 should be written

$$\alpha d = \arctan\left[\frac{\gamma}{\alpha}\right] + \arctan\left[\frac{\delta}{\alpha}\right] + m\pi \quad , \tag{30}$$

where m = 0,1,2,3, ... is the mode number.

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Substituting the expressions for E_y (Equation 24) into the wave equation (Equation 20), yields

$$\delta^2 + k^2 n_0^2 - \beta^2 = 0 \quad , \tag{31}$$

$$-\alpha^2 + k^2 n_1^2 - \beta^2 = 0 \quad , \tag{32}$$

$$\gamma^2 + k^2 n_2^2 - \beta^2 = 0 \quad , \tag{33}$$

where $k = 2\pi/\lambda_0$, and eliminating β from the equations we get,

$$\delta^2 + \alpha^2 = k^2(n_1^2 - n_0^2) \quad , \tag{34}$$

$$\gamma^2 + \alpha^2 = k^2(n_1^2 - n_2^2) \quad . \tag{35}$$

These two equations, together with Equation 30, are the complete set of equations which determine the TE modes. The above equations can be applied to either an asymmetric or a symmetric guide. First, the equations will be solved for the asymmetric case. For a typical asymmetric guide, the refractive indices n_0, n_1 and n_2 satisfy the relationship that $n_1 >> n_0$ and $n_1 \approx n_2$. This would be true in most cases when the top boundary layer is air ($n_0 = 1$). The simplest way to solve these equations is to solve them graphically. Relations 34 and 35 are equations of a circle with radii of

$$R_1 = (\delta^2 + \alpha^2)^{\frac{1}{2}} \tag{36}$$

and

$$R_2 = (\gamma^2 + \alpha^2)^{\frac{1}{2}} \quad , \tag{37}$$

and they are shown in Figure 3. For the indices of $n_0 = 1$, $n_1 = 1.55$ and $n_2 = 1.52$, the circle describing $\delta^2 + \alpha^2$ would be approximately 15 times larger than $\gamma^2 + \alpha^2$. As can be seen from Figure 3, the range of values which α and γ can take on are the same and are much larger than range of δ. However, the magnitude of δ is much larger than the magnitudes of α and γ. This will simplify Equation 30 by using $\delta/\alpha >> 1$ which makes arc tan $\delta/\alpha \approx \pi/2$ and leads to

$$\alpha d = \text{arc tan}\left[\frac{\gamma}{\alpha}\right] + \pi/2 + m\pi \quad , \tag{38}$$

which can be rewritten in the form

$$\gamma d = \alpha d \tan(\alpha d - \pi/2 - m\pi) = -\alpha d \cot \alpha d \quad . \tag{39}$$

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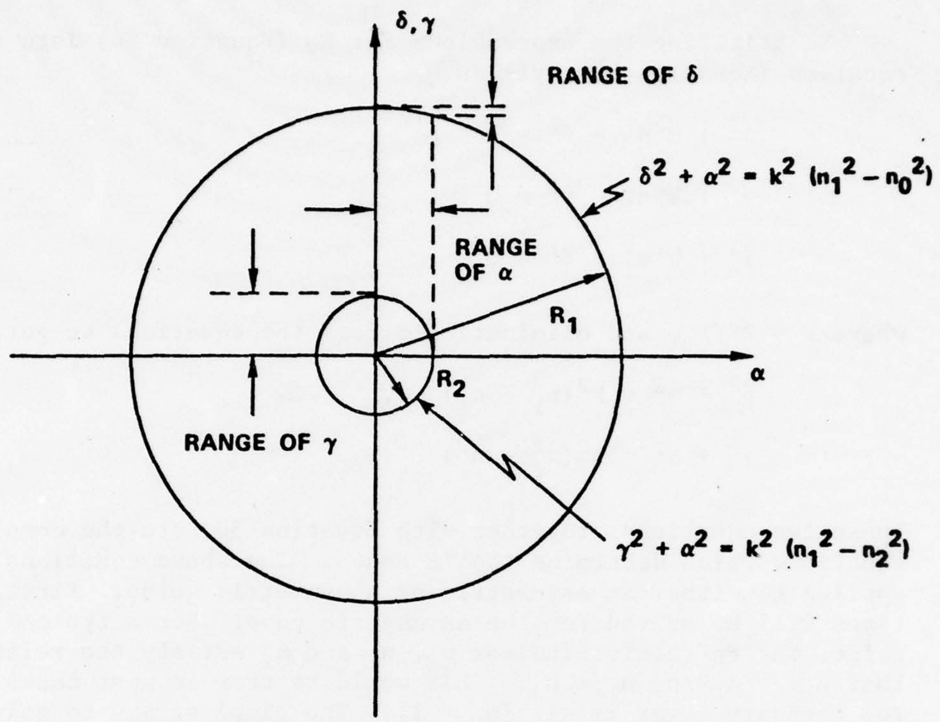

FIGURE 3. Diagram Which Illustrates the Relative Magnitude of α, δ and γ.

Equation 39 and

$$(\gamma d)^2 + (\alpha d)^2 = (kd)^2(n_1^2 - n_2^2) \tag{40}$$

are each plotted in Figure 4 and the intersection of the two curves are the solutions. As discussed earlier, γ must be positive to have guided modes; consequently, only the first quadrant is plotted for the circles. The intersections where γ is negative correspond to the even modes; therefore, only odd modes exist for the asymmetric waveguide. It can be seen that asymmetric dielectric waveguides have a low frequency (long wavelength) cutoff below which energy does not propagate. For a particular propagating mode the values of γ and α are determined by the circle of radius R where

$$R = kd(n_1^2 - n_2^2)^{\frac{1}{2}} \quad , \tag{41}$$

and for the mth propagating mode the radius of the circle must be between

$$(m + 1)\pi \geq kd(n_1^2 - n_2^2) \geq (2m + 1)\pi/2 \quad . \tag{42}$$

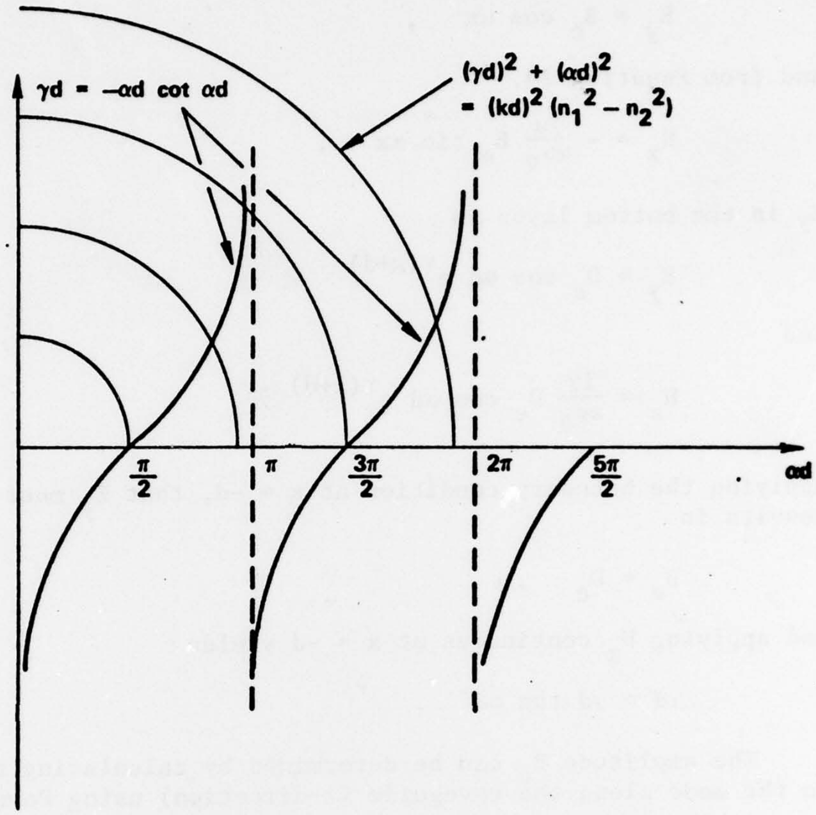

FIGURE 4. Graphical Solutions of the Transcendental Equations for the Asymmetric Waveguide.

The condition of index change $\Delta n = n_1 - n_2$ for cutoff of the m^{th} mode becomes

$$\Delta n = \frac{(2m - 1)^2}{32n_2}\left[\frac{\lambda}{d}\right]^2 \quad , \tag{43}$$

where λ is the free space wavelength and d is the waveguide thickness. The number of modes that propagate are finite and are dependent on the parameters n_1, n_2, λ and d which are controllable. For example, keeping λ/d constant, an index change by a factor of nine is required for the second mode to propagate over that required for the lowest order mode.

If the dielectric waveguide is symmetric, i.e., $n_2 = n_0$, then some surprisingly different results are obtained. The analysis is simplified if the modes are separated into even and odd modes due to the symmetry. Considering even modes first, from Equation 24, E_y in the waveguide becomes (subscript e is for even)

$$E_y = B_e \cos \alpha x \quad , \tag{44}$$

and from Equation 19,

$$H_z = -\frac{i\alpha}{\omega\mu_0} B_e \sin \alpha x \quad . \tag{45}$$

E_y in the bottom layer is

$$E_y = D_e \cos \alpha d \, e^{\gamma(x+d)} \tag{46}$$

and

$$H_z = \frac{i\gamma}{\omega\mu_0} D_e \cos \alpha d \, e^{\gamma(x+d)} \quad . \tag{47}$$

Applying the boundary condition at x = -d, that E_y must be continuous results in

$$B_e = D_e \quad , \tag{48}$$

and applying H_z continuous at x = -d yields

$$\gamma d = \alpha d \tan \alpha d \quad . \tag{49}$$

The amplitude B_e can be determined by calculating the power flowing in the mode along the waveguide (z-direction) using Poynting's theorem

$$P = \frac{1}{2} \int_{-\infty}^{\infty} (\vec{E} \times \vec{H}^*)_z \, dx \quad . \tag{50}$$

Since the above involves a cross product and the only component of a TE mode is E_y, then the magnetic field component needed is H_x; therefore,

$$P = -\frac{1}{2} \int_{-\infty}^{\infty} E_y H_x^* \, dx \quad , \tag{51}$$

and from Equation 18,

$$H_x = -\frac{\beta}{\omega\mu_0} E_y \quad , \tag{52}$$

which gives

$$P = \frac{\beta}{\omega\mu_0} \int_{-\infty}^{\infty} |E_y|^2 \, dx \quad . \tag{53}$$

Substituting Equations 44 and 46 into Equation 53, and using Equation 49, yields

$$B_e = \left[\frac{2\omega\mu_0 P}{\beta d + \beta/\gamma}\right]^{\frac{1}{2}} . \qquad (54)$$

Similarly, the odd modes are solved in the same manner using

$$E_y = C_0 \sin \alpha x \qquad (55)$$

inside the waveguide with

$$H_z = \frac{i\alpha}{\omega\mu_0} C_0 \cos \alpha x \quad . \qquad (56)$$

In the bottom layer,

$$E_y = -E_0 \sin \alpha d \; e^{\gamma(x+d)} \qquad (57)$$

and

$$H_z = \frac{-i\gamma}{\omega\mu_0} E_0 \sin \alpha d \; e^{\gamma(x+d)} \quad , \qquad (58)$$

where the minus sign comes from the fact that it is an odd mode. Using the continuity of the fields across the boundary yields

$$C_0 = E_0 \qquad (59)$$

and

$$\gamma d = -\alpha d \cot \alpha d \quad . \qquad (60)$$

The power flow calculation yields

$$C_0 = \left[\frac{2\omega\mu_0 P}{\beta d + \beta/\gamma}\right]^{\frac{1}{2}} . \qquad (61)$$

One has to be careful when using Equations 54 and 61 since they look the same. When using the relation for B_e, the constants β and γ are evaluated from relation 49; whereas, relation 60 is used to evaluate β and γ for C_0.

Figure 5 shows a plot of relations 49 and 60, and relation 40 is used in the same manner to find the graphical solutions as was done for the asymmetric waveguide. As can be seen from the solutions of the transcendental equations for the symmetric waveguide, there is no low frequency cutoff for the lowest order propagating mode. The frequency can be made arbitrarily low (smaller radius circle) and a solution still exists (see Figure 5).

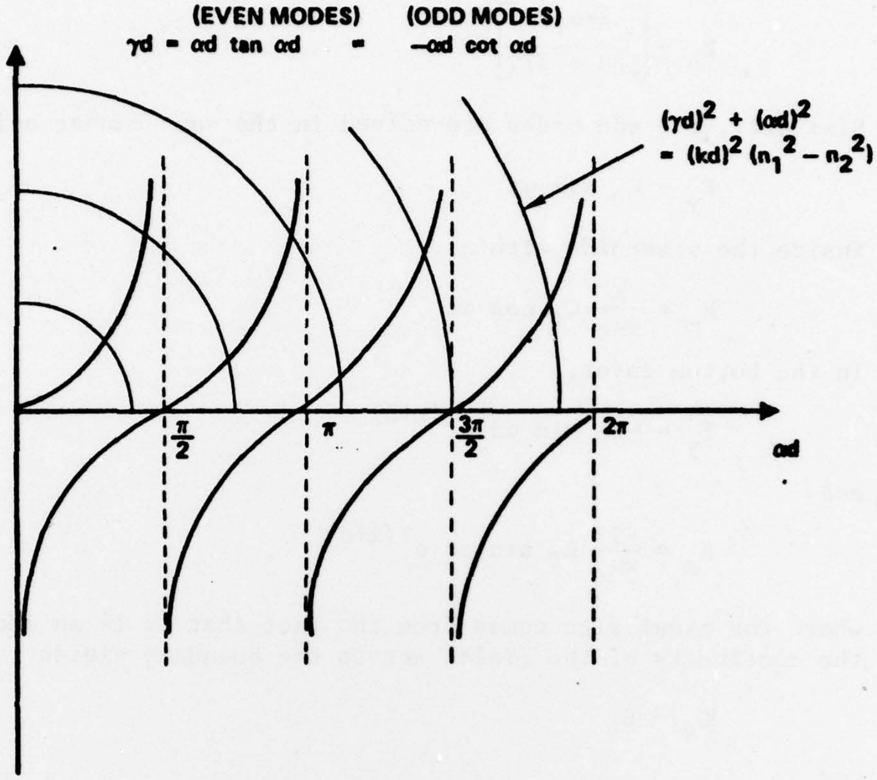

FIGURE 5. Graphical Solutions of the Transcendental Equations for the Symmetric Waveguide.

Another important aspect of guided wave optics is the dispersion in the waveguides, i.e., how the propagation constant, β, varies with frequency. The dispersion relationship is obtained by substituting the equations for α, γ and δ (Equations 21-23) into Equation 30, which yields

$$[(k^2n_1^2 - \beta^2)d^2]^{\frac{1}{2}} = \arctan\left[\frac{(\beta^2 - k^2n_2^2)^{\frac{1}{2}}}{(k^2n_1^2 - \beta^2)^{\frac{1}{2}}}\right] + \arctan\left[\frac{(\beta^2 - k^2n_0^2)^{\frac{1}{2}}}{(k^2n_1^2 - \beta^2)^{\frac{1}{2}}}\right] + m\pi \tag{62}$$

for TE modes of an asymmetric waveguide. Also, remember for guided modes, the propagation constant obeys

$$kn_0 < kn_2 < \beta < kn_1 \quad , \tag{63}$$

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where $k = \omega/c$. Therefore, as can be seen in Figure 6, β is bounded by $\omega/c\ n_1$ above which no solutions exist, and $\omega/c\ n_2$ below which exist radiation modes. Figure 6 also shows what three typical dispersion equations look like, and Figure 7 shows computer solutions obtained for an asymmetric waveguide with $n_0 = 1$, $n_1 = 2.23$, $n_2 = 1.393$ and $d = 3.34$ µm.[9] Physically, the shape of the dispersion curves is explained as follows. Increasing the frequency, the depth of penetration of the exponential tails of the fields into the surrounding media decreases, which means that the electric fields are more confined to the guiding layer. In the limit as $\omega \to \infty$ the fields are totally confined to the waveguide layer. The slope of the upper boundary is the velocity of light in the substrate with index n_2. When ω/c approaches kn_2 the constant γ approaches zero and this causes the exponential tail of the field to extend infinitely into the substrate.

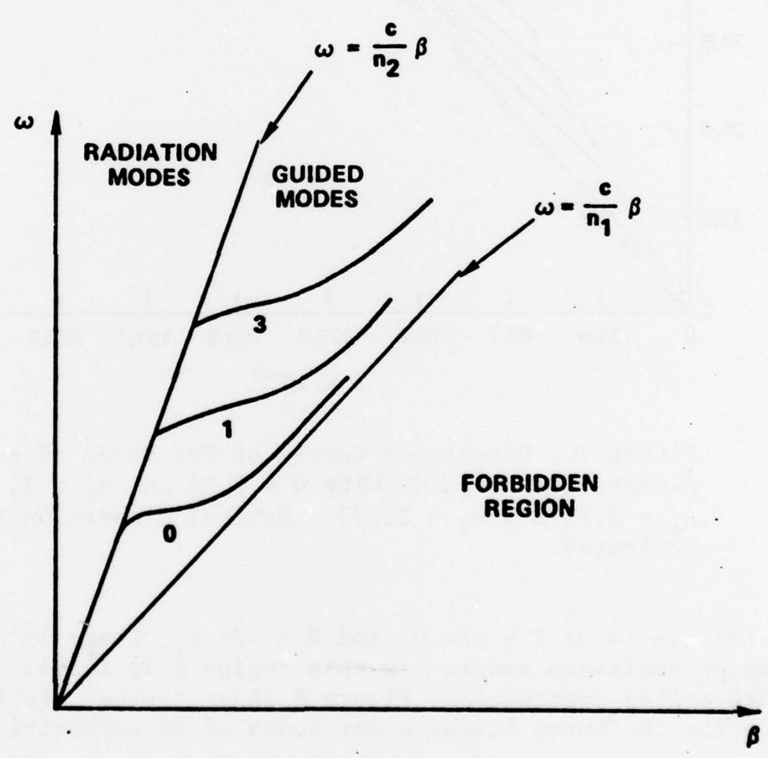

FIGURE 6. Typical ω Versus β Diagram Showing the Dispersion Relationship of Three Discrete Guided Modes for Dielectric Slab Waveguides.

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[9] M. K. Barnoski. *Introduction to Integrated Optics*. New York, Plenum Press, 1974.

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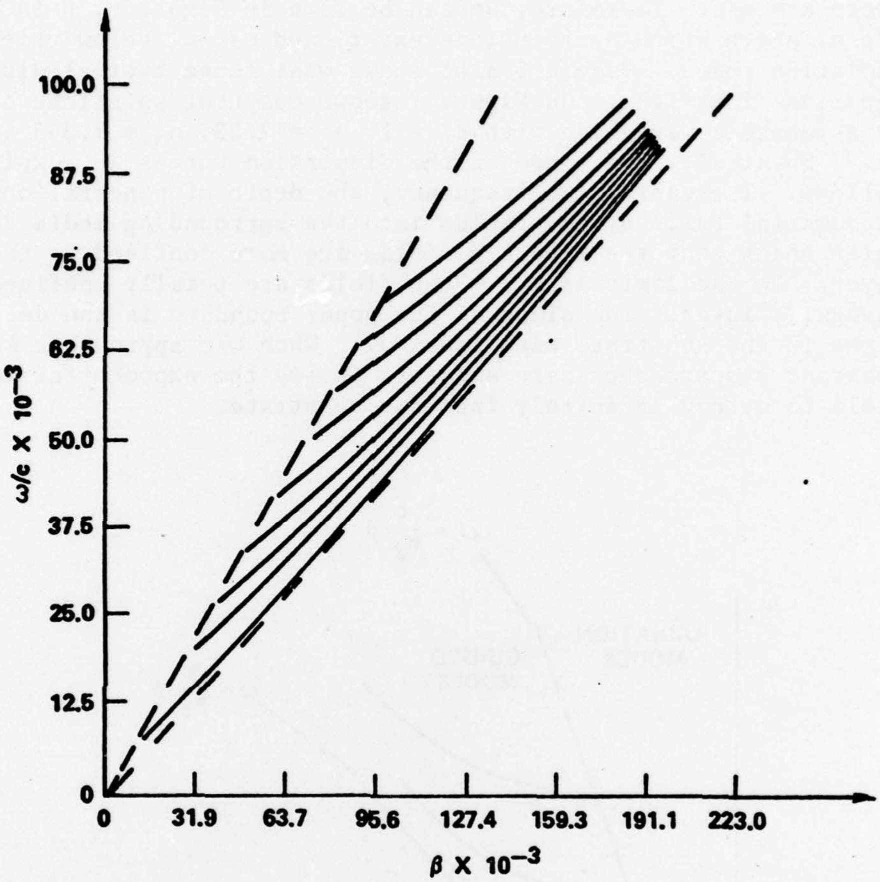

FIGURE 7. Dispersion Curves of Ten Modes of an Asymmetric Waveguide With d = 3.34 μm, $n_0 = 1$, $n_1 = 2.23$ and $n_2 = 1.393$. Natural dispersion is neglected.

Beyond the limits of $\beta = \omega/c\ n_2$ and $\beta = \omega/c\ n_1$, there is a continuous spectrum of radiation modes. In this region β is imaginary and these modes are called evanescent. Figure 8 shows typical electric field patterns for the three lowest order modes of an asymmetric waveguide.

This concludes the discussion for the TE-guided modes of a dielectric waveguide. The analysis for TM modes is very similar except the field components are E_x, E_z and H_y. The TM modes are obtained by setting $H_z = 0$. The resulting transcendental equations are much the same. The equations of the circles are identical to those for the TE modes (i.e., relations 34 and 35). The difference between TE and TM modes is in Equation 30, where the ratio γ/α in the TE case becomes $(n_1/n_2)^2\ \gamma/\alpha$ for TM modes and the ratio δ/α in the TE case becomes $(n_1/n_0)^2\ \delta/\alpha$ for TM modes.

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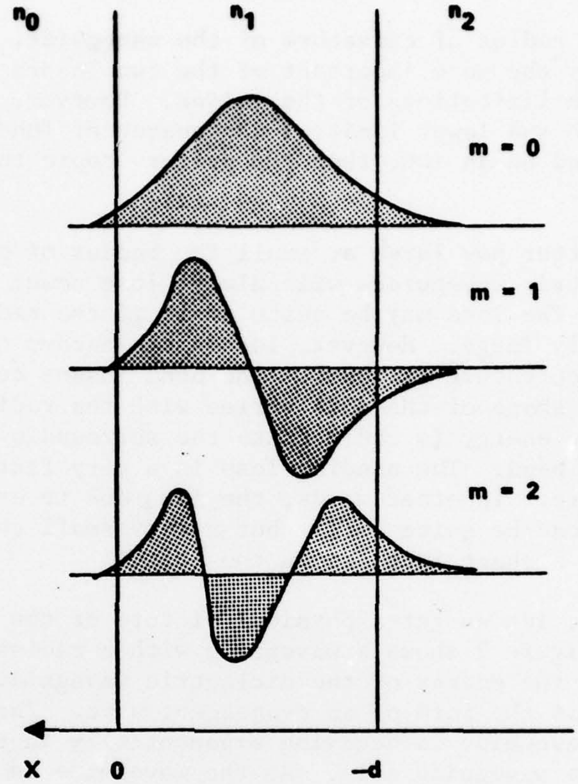

FIGURE 8. Typical Electric Field Intensity Patterns for Three Lowest Order Modes.

Now we have a useful understanding of guided wave optics. How the parameters of the waveguide can be changed to affect the propagating modes is understood. This treatment can be extended to two dimensions (i.e., rectangular dielectric waveguides) but the results are much the same except guiding takes place in both of the transverse directions. In deriving the wave equation, $\partial/\partial y$ can no longer be taken as zero. At any rate, now we can consider other important topics by using our knowledge of the waveguide modes. The next section will consider losses in dielectric waveguides due to bending of the waveguides. These losses are important in establishing the maximum packing density of integrated optical circuits (IOCs).

BENDING LOSSES

There are many factors which limit the performance of waveguides in IOCs. Two of the more important ones are scattering losses which are due to waveguide edge roughness, and bending losses which are due

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to a short radius of curvature of the waveguide. Bending losses is not necessarily the more important of the two losses in describing the performance limitations of the guides. However, if one is to be concerned with the lower limit of the number of functions per unit area to be performed on an IOC, then the primary topic to be considered is bending losses.

No matter how large or small the radius of curvature of the bend, the dielectric waveguides will always lose power by radiation around the bend. The loss may be quite small if the radius of curvature is sufficiently large. However, losses can become quite large if the radius of curvature is small. The bend losses come from two effects: First, the shape of the mode varies with the radius of curvature, and second, the energy is radiated to the surrounding media as it travels around the bend. The bending loss is a very fast function of the radius of curvature. In other words, the loss due to some value of radius of curvature can be quite small, but a very small change in the bend radius will cause a sharp increase in the loss.

First, let us get a physical picture of the causes of the bending losses. Figure 9 shows a waveguide with a radius of curvature R. A portion of the energy of the dielectric waveguides travels outside the waveguide in the form of an evanescent wave. The evanescent field outside the waveguide is decaying exponentially in the transverse direction to the waveguide axis. As the waveguide is bent the planes of constant phase are as shown in Figure 9. The planes of constant phase are separated further apart the farther one goes from the center of curvature. Therefore, the farther from the center of curvature the wave is, the faster it must travel so that it can still be in phase with the guided wave. Each medium has its own characteristic phase velocity which is determined by the index. The surrounding media has a larger phase velocity than does the waveguide since the index is smaller. At some critical distance from the waveguide, the evanescent wave must travel faster than its characteristic phase velocity to remain in phase with the rest of the wave. Since this is not a physically realizable situation the wave detaches itself from the rest of the wave and radiates into space. The sharper the bend of the waveguide, the closer the critical distance is to the waveguide. Since the evanescent wave is an exponentially decaying wave the field increases exponentially towards the waveguide and, as will be shown, the critical distance varies linearly with the radius of curvature. Therefore, only a small change in the radius can make an exponential change in the field that is radiated away from the waveguide.

Figure 10 shows a propagating mode along with its associated evanescent wave in an expanded view of a curved waveguide. The angle θ is the angle between adjacent planes of constant phase. At the critical distance $(R + r)$, the distance that the evanescent wave must travel from one plane of constant phase to the next is $(R + r)\theta$. The distance that

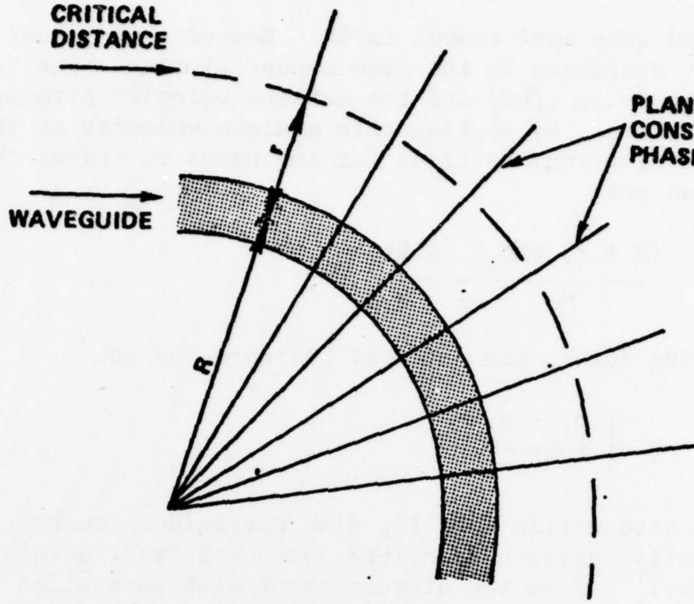

FIGURE 9. Curved Slab Waveguide With Radius of Curvature R and Critical Distance r (Distances Measured From Center of Waveguide). Straight lines are planes of constant phase.

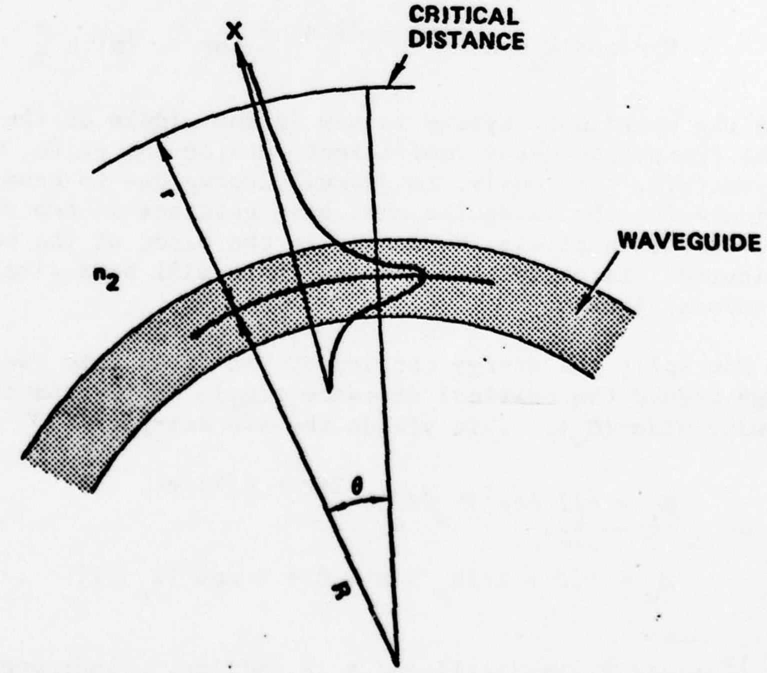

FIGURE 10. Expanded View of the Curved Waveguide.

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the guided wave must travel is $R\theta$. However, they must travel these different distances in the same amount of time. The velocity inside the waveguide is ω/kn_1 and the maximum velocity permissible in the outer media is ω/kn_2. By setting this maximum velocity at the critical distance and equating the times for the waves to travel the different distances, we get

$$\frac{(R + r)\,\theta kn_2}{\omega} = \frac{R\theta kn_1}{\omega} \quad , \tag{64}$$

and solving for r, the critical distance, we get

$$r = \left[\frac{n_1 - n_2}{n_2}\right] R \quad . \tag{65}$$

The attenuation loss for slab waveguides can be calculated using the velocity approach described above and first developed by Marcatili and Miller.[10] From the discussion of slab waveguides in the previous section, the transverse field distribution can be described as

$$E = \cos(k_x x) \qquad \text{for} \quad \frac{-d}{2} \le x \le \frac{d}{2}$$

and

$$E = \cos(k_x d)e^{-[|x| - d]/\eta} \qquad \text{for} \quad |x| \ge \frac{d}{2} \quad , \tag{66}$$

where the coordinate system is now in the middle of the waveguide, η^{-1} is the transverse decay coefficient outside the guide, and the waveguide is symmetric. Obviously, to discuss losses due to bends, as described by Figure 10, the waveguide must have guidance in two dimensions. However, in this analysis the losses in the plane of the waveguide will be considered. Later in this section there will be a discussion of two-dimensional losses.

Now split the energy carried by the field into two parts: (1) that energy beyond the critical distance $r(\mathcal{E}_r)$ and (2) the total energy in the waveguide ($\mathcal{E}_w$). This yields the two energies[10]

$$\mathcal{E}_r = \eta/2 \cos^2[k_x d/2]e^{-2[r - b/2]/\eta} \tag{67}$$

and

$$\mathcal{E}_w = \eta/2 + 1/2k_x \sin k_x d + \eta \cos^2[k_x d/2] \quad . \tag{68}$$

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[10] E. A. J. Marcatili and S. E. Miller. "Improved Relations Describing Directional Control in Electromagnetic Waveguidance," BELL SYST TECH J, Vol. 48 (September 1969), pp. 2161-88.

Now let P_o be the propagating power and α the attenuation constant where

$$\alpha = \frac{1}{2P_o} \frac{dP}{dz} \quad . \tag{69}$$

Let a be the transverse field width, then ℓ, the distance that the energy remains collimated in an infinite medium, is given by

$$\ell = \frac{a^2}{2\lambda} \quad , \tag{70}$$

and the use of the relations for the energy (Equations 67 and 68) yields

$$\ell = \frac{[d + 2\eta\cos(d_x d/2)]^2}{2\lambda} \quad . \tag{71}$$

The attenuation constant α in terms of the energy is given by

$$\alpha = \frac{1}{2\ell} \frac{\mathcal{E}_r}{\mathcal{E}_w} \quad . \tag{72}$$

Substituting in the relations for r, ℓ, $\mathcal{E}_r$ and $\mathcal{E}_w$ yields

$$\alpha = A\, e^{-BR} \quad , \tag{73}$$

where

$$A = \frac{[\lambda\eta/2]\cos^2(k_x d)e^{\eta b/2}}{[\eta/2 + (1/2k_x)\sin k_x d + \eta\cos^2(k_x d/2)][d + 2\eta\cos(k_x d/2)]} \tag{74}$$

and

$$B = \frac{2}{\eta}\left[\frac{n_1 - n_2}{n_2}\right] \quad . \tag{75}$$

These results further show the commanding influence that the bending radius has on the loss. Table 1 shows some results[10] using the above analysis with the waveguide index of refraction = 1.5. Figure 11 shows a plot of the loss as a function of R for case 2. Note that a 40% decrease in R brings about two orders of magnitude increase in the loss.

Marcatili has made a complete analysis of the losses in rectangular dielectric waveguides.[11] In his analysis he used a modal analysis as

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[11] E. A. J. Marcatili, "Bends in Optical Dielectric Guides," BELL SYST TECH J, Vol. 48 (September 1969), pp. 2103-32.

TABLE 1. Waveguide Losses as a Function of Radius of Curvature.

Case	d, μm	Index of surrounding medium	A, nepers/m	B, meters^{-1}	R for α=1 neper/m
1	0.198	1.0	2.57×10^6	3.47×10^6	4.25 μm
2	1.04	1.485	1.04×10^5	1.46×10^4	0.79 mm
3	1.18	1.4985	5.4×10^3	81.4	0.106 m

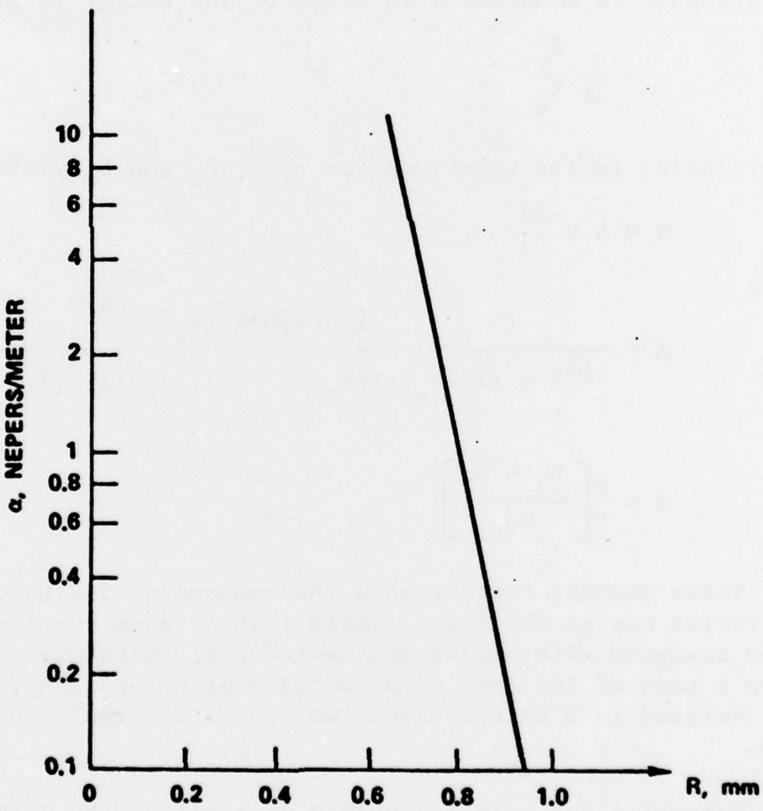

FIGURE 11. Radiation Loss Versus Bend Radius.

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opposed to the velocity approach described above. Since his paper has such a complete analysis, and his results are very similar to those just obtained, this discussion will just present some of the important results not covered in the velocity analysis.

In the modal analysis, the effect of guide curvature is to change the real part of the propagation constant, introduce an imaginary part of the propagation constant corresponding to the loss, and alter the field distribution. This also leads to the radiation loss being related to the bend radius exponentially. Figure 12 shows the field distribution as a function of guide width, d. It also shows that the field distribution perpendicular to the plane of curvature is symmetric and unchanged due to bending of the waveguide.

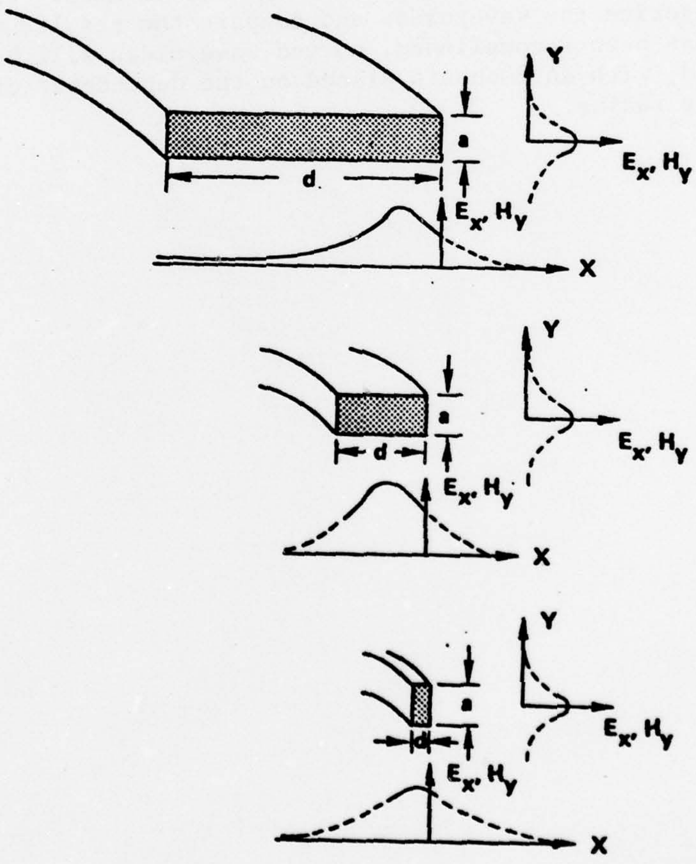

FIGURE 12. Field Distribution as a Function of Guide Width d. All three waveguides have the same radius of curvature.

DISCUSSION

It has been shown that losses in dielectric optical waveguides due to bends in the waveguides can be quite large. When considering packaging density of integrated optical circuits, bending losses definitely need to be taken into account. The losses are an exponential function of the bending radius.

Further investigation of the bending losses needs to be performed. In particular, experimental studies are needed. A number of theoretical aspects of bending losses have been reported; however, no experimental investigations have yet been reported. The next step of this program is to investigate the experimental aspects of bending losses. A program is now underway to make diffused waveguides in $LiNbO_3$ and to experimentally characterize the waveguides and compare the results with theory. Once this has been accomplished, curved waveguides will be made and characterized, with an emphasis placed on the dependence of bending loss on bending radius.

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